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Resolved gravity duals of $\mathcal{N} = 4$ quiver field theories in $2 + 1$ dimensions

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ABSTRACT: We generalize the construction by Aharony, Hashimoto, Hirano, and Ouyang of $\mathcal{N} = 4$ quiver gauge theory with gauge group $U(N + M) \times U(N)$, k fundamentals charged under $U(N)$ and bi-fundamentals, to the case with gauge group $\prod_{i=1}^{\hat{k}} U(N_i)$ with k_i fundamentals charged under $U(N_i)$. This construction is facilitated by considering the resolved $ALE_{\hat{k}} \times TN_k$ background in M-theory including non-trivial fluxes through the resolved 4-cycles in the geometry. We also describe the M-theory lift of the IIA Page charge quantization condition. Finally, we clarify the role of string corrections in various regimes of parameter space.

KEYWORDS: AdS-CFT Correspondence, Brane Dynamics in Gauge Theories, Gauge-gravity correspondence

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1 Introduction

$\mathcal{N} = 4$ gauge field theories in $2 + 1$ dimensions is a rich dynamical system, exhibiting features such as quantum corrected moduli spaces and enhancons in the holographic dual. The number of supersymmetries is the same as theories in $3 + 1$ dimensions with $\mathcal{N} = 2$ supersymmetry, whose vacuum structure can be analyzed exactly using the techniques of Seiberg and Witten [1, 2]. The relation between $\mathcal{N} = 2$ theories in $3 + 1$ dimensions and $\mathcal{N} = 4$ theories in $2 + 1$ dimensions is a rich subject on its own [3]. One complication stems from the fact that $\mathcal{N} = 4$ vector multiplets in $2 + 1$ dimensions have twice as many scalar components compared to $\mathcal{N} = 2$ vector multiplets in $3 + 1$ dimensions. There are, however, other powerful tools at our disposal to explore the quantum dynamics of $\mathcal{N} = 4$ theories in $2 + 1$ dimensions, such as mirror symmetry [4], localization, and holography. Recently, the structure of Coulomb branch of $\mathcal{N} = 4$ theories in $2 + 1$ dimensions was mapped out formally in [5].

The fact that full quantum dynamics is accessible on the field theory side provides an opportunity to explore subtle issues in gauge/gravity duality where string and quantum corrections are expected on the gravity side of the correspondence. Knowing the existence, and in some instance, the analytic form, of certain physical quantities on the field theory side provides a concrete target that one can aim to reproduce on gravity side.

The aim of this paper is to lay the foundation for such analysis. For a particular class of $\mathcal{N} = 4$ theories in $2 + 1$ dimensions with gauge and matter content illustrated in figure 1a, a gravity dual in type IIA was constructed explicitly and was analyzed in some detail by Aharony, Hashimoto, Hirano, and Ouyang (AHHO) in [6] and more recently by Cottrell, Hanson, and Hashimoto (CHH) in [7]. In figure 1b, we illustrate the brane construction for these models in type IIB string theory. In the brane construction, one first constructs a defect field theory in $3 + 1$ dimensions on $R^{1,2} \times S^1$. One recovers the theory

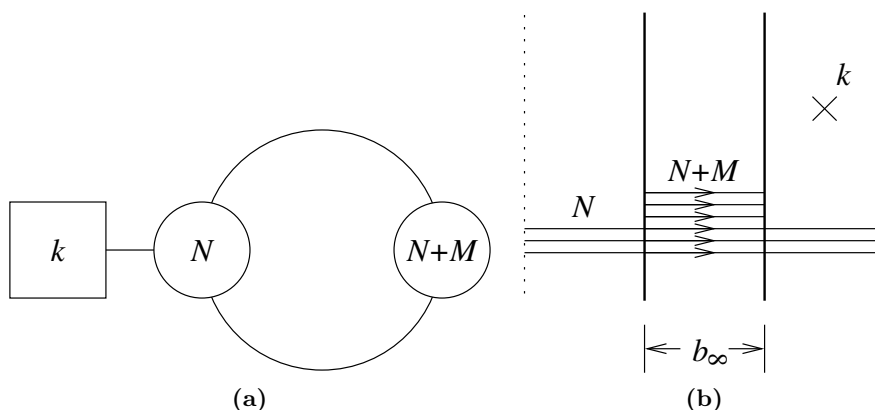


Figure 1. (a) Quiver diagram of $U(N + M) \times U(N)$ with k flavors. (b) Brane construction for the theory studied in AHHO and CHH. We have M fractional D3 branes suspended between two NS5 branes in addition to N integer branes. The horizontal direction is periodically identified at the dotted lines.

in 2+1 dimensions by taking the radius of S^1 to zero keeping the gauge coupling in 2+1 dimensions fixed.

These theories are formulated as $U(N + M) \times U(N)$ gauge field theories with k flavors charged under $U(N)$ and two bifundamental matter fields in the ultra-violet. Such a system will then undergo a renormalization group flow. The structure in the IR will depend on the gauge group and the matter content. In [8] Gaiotto and Witten classified the IR dynamics of broad class of $\mathcal{N} = 4$ theories in 2+1 dimensions into categories “good”, “ugly”, and “bad”. The “good” theories flows in the IR to interacting superconformal fixed point. The “bad” theories appear to exhibit a more more complicated flow that depends sensitively on details of the model [9, 10]. For the class of models illustrated in figure 1a, the condition for the theory to be “good” is

$$0 \leq -\min(M, 0) \leq N \quad (1.1)$$

and

$$0 < -2M < k . \quad (1.2)$$

The first condition (1.1) is simply the requirement that the rank of each gauge group in $U(N + M) \times U(N)$ is positive. The second is the condition for the gauge coupling not to diverge in the RG flow.

It would be interesting and desirable to identify the manifestation of conditions (1.1) and (1.2) in the gravity dual. This very issue was studied recently in [7], which provided the following general picture. A supergravity ansatz can be set up, and solved, for any choice of N , M , and k satisfying

$$N - \frac{M^2}{k} \geq 0 . \quad (1.3)$$

Violating this bound will give rise to a repulson singularities. For the set of repulson free solutions satisfying (1.3), one can explore the possibility of various brane probes becoming tensionless. If that happens, we say that “the background suffers from enhancon

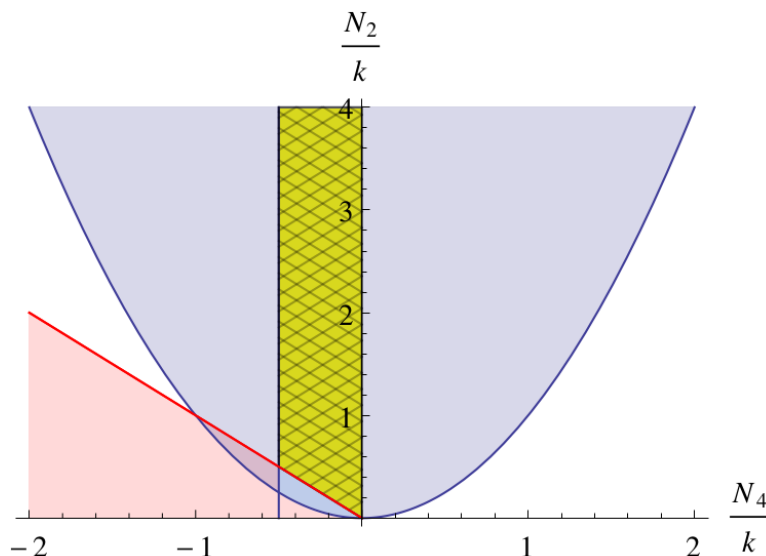


Figure 2. One may partition the set of field theories according to various criteria. Here, the parabola indicates theories with positive brane charge, the yellow cross-hatched strip labels the “good” theories and the red wedge region is the parts excluded by field theory considerations.

effects [11]”. The analysis of [7] revealed that the condition for the enhancon not to appear is precisely (1.2).

The remaining issue, then, is how to properly understand the apparent discrepancy between (1.1) and (1.3). A useful way to highlight this issue is to work in the scaling limit

$$k \rightarrow \infty, \quad \frac{N}{k} = \text{fixed}, \quad \frac{M}{k} = \text{fixed}. \quad (1.4)$$

This is somewhat like working in the ’t Hooft limit for these models. The features expected from the gauge theory perspective and the gravity perspective for various choice of N and M is illustrated in figure 2.

Before proceeding, let us note that scaling N and M like k implies

$$N \ll k^5 \quad (1.5)$$

and as such, we must use the type IIA description over the M-theory description. The curvature radius in type IIA description then scales as $R^2 \sim \alpha' \sqrt{N/k}$. These were the scaling found in [12] but they are applicable for our setup as well.

The proper understanding of the issue of (1.1) v.s. (1.3) which emerges is as follows. For the set of values outside (1.2), the supergravity solution contains an enhancon. As such, the issue of whether there is or isn’t a repulson at (1.3) is meaningless. What we are saying is that inferring (1.1) on the gravity side outside (1.2) requires *resolving* the dynamics of the enhancon. On the other hand, for the models inside the range (1.2), the values of N/k and M/k all take values of order one when either (1.1) or (1.2) are saturated. This then suggests that (1.3) receives α' corrections which when properly resummed reproduces (1.1). In other words, the apparent discrepancy between (1.1) and (1.3) can be attributed entirely to α' correction at least in the scaling regime (1.4).

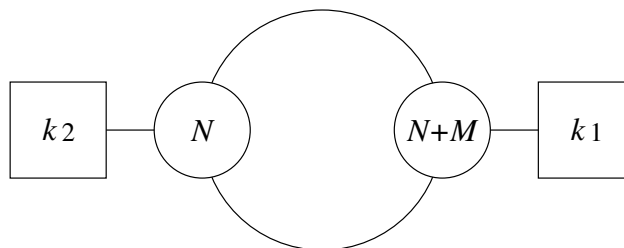


Figure 3. Quiver diagram for a theory with flavors charged under both gauge groups.

Now that we have a better understanding of the relationship between (1.1) and (1.3), there are a number of interesting directions one can explore. For the “good” theories satisfying (1.2), one can use localization techniques [13, 14] to compute the free energy exactly. It would be interesting to recover (1.3) as the leading large N/k approximation as well as subleading terms and compare the first few corrections to curvature corrections on the gravity side.

For theories not in the range (1.2), the task of resolving the enhancon seems quite daunting. On the other hand, we know from previous studies on related systems in 3+1 dimensions [15] that these enhancons are closely associated with the locus of enhanced gauge symmetry on the Coulomb branch as well as being the baryonic root, a point on the Coulomb branch from which the Higgs branch emanates [16]. More detailed analysis of string correction in gauge gravity correspondence of $\mathcal{N} = 2$ systems in 3+1 dimensions were carried out in [17, 18]. It would be very interesting to understand how the version of this story in 2+1 dimensions is manifested on the gravity side of the gauge/gravity correspondence.

One way in which one might imagine approaching the baryonic root is to consider turning on FI parameters. In gauge theory FI parameters generically smooth out the origin of Higgs branch and lift the Coulomb branch. One can then approach the baryonic root by studying the limit in which FI parameter is taken to zero.

In order to fully explore the relationship between the field theory and the gravity formulation of these $\mathcal{N} = 4$ systems, it would also be useful to have access to more general construction than the class of models covered in figure 1a. One obvious generalization is to allow matter to be charged under $U(N + M)$ as well as $U(N)$, i.e. to consider a quiver of the type illustrated in figure 3.

It is not too difficult, it turns out, to generalize the construction of the gravity solution to the one corresponding to the brane construction illustrated in figure 4 with generic FI and mass parameters turned on.

The goal of the remainder of this note is to describe such a construction. As we will describe in detail below, considering generic FI and quark mass naturally leads to the generalization of the ansatz to include the possibility of adding matter charged under different components of the product gauge group. One can then consider the limit of vanishing FI and mass parameters and obtain a candidate gravity dual for the field theory with vanishing FI and mass parameters at least for the particular point on the Coulomb branch that one reaches in this limiting procedure.

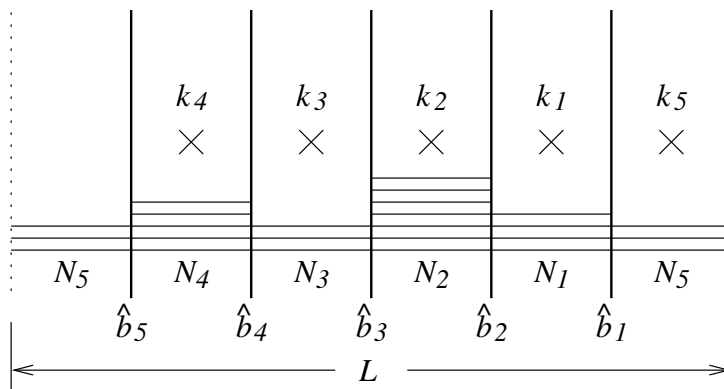


Figure 4. General circular quiver.

One of the new results which we present in this article is the explicit mapping between discrete field theory parameters and the quantized fluxes threading various cycles which arises in our construction on gravity side. The details of the mapping is somewhat intricate. We summarize our notation for the data on the field theory side in section 3.1 and related them to gravity data in equation (4.30). This analysis was facilitated by turning on the generic mass and FI parameters which regularized the geometry on the gravity side. This makes the analysis of charge quantization much more straightforward and provides an independent derivation of seemingly exotic charge and flux quantization relations outlined in [6].

An important lesson we draw from these analyses is the fact that α' corrections play a critical role in characterizing the behavior of these systems close to the threshold of saturating the conditions necessary for unbroken supersymmetry. It seems likely that this is a generic fact about gauge/gravity duality and implies that understanding full string dynamics is required in order to study phenomena such as dynamical supersymmetry breaking in gauge/gravity duality [19, 20]. It would be very interesting to distill this issue and identify a tractable string theory model which captures the dynamics of supersymmetric field theories near the threshold of dynamical supersymmetry breaking holographically, possibly along the lines of [21].

The organization of this paper is as follows. In section 2 we review the basic setup and results of AHHO and CHH. In section 3 we describe the general brane construction of the theories we are interested in as well as the mapping to field theory. Next, in section 4, we show how to obtain sugra solutions corresponding to these brane diagrams in M-Theory. In section 5 we also describe the IIA reduction and verify certain aspects of the gauge/gravity duality using a probe analysis. Finally, we offer our conclusions.

Some aspect of our supergravity construction, particularly the enumeration of fluxes through compact cycles of the *ALE* background geometry, can be found also in the previous work of [22]. In our treatment, we provide additional consideration of fluxes threaded by the non-compact cycles of the *ALF* which is related parametrically to the coupling constants of the 2+1 dimensional gauge theory in the UV. We also elaborate on the quantization of fluxes and charges from the IIA and M-theory perspective, and highlight our expectations on correction due to α' effects.

2 Review of AHHO and CHH

In [6], the authors considered a class of theories consisting of $\mathcal{N} = 4$ SYM in 2+1 dimensions with gauge group $U(N + M) \times U(N)$ and k fundamental hypermultiplets charged under $U(N)$. They are represented by a circular quiver of the form illustrated in figure 1a. Such a model can be constructed from the type IIB brane configuration illustrated in figure 1b. The construction involves 2 NS5-branes and k D5-branes, N “integer” D3-branes winding all the way around the S^1 of period L , and M “fractional” D3-branes suspended between the two NS5-branes separated by the distance $b_\infty L$. In the $\alpha' \rightarrow 0$ zero slope limit, most of the string states decouple and we obtain a 3+1 dimensional defect theory on $R^{1,2} \times S^1$. In the limit that L goes to zero while keeping the gauge coupling in 2+1 dimensions fixed, momentum modes along the S^1 decouples and we obtain a theory in 2+1 dimensions.

The gravity dual is constructed by T-dualizing along S^1 which maps the 2 NS5-branes to TN_2 (which approaches the $\mathbb{C}^2/\mathbb{Z}_2$ ALE geometry in the $L \rightarrow 0$ limit), D5-branes to D6-branes, integer D3-branes to D2-branes, and fractional D3-branes to fractional D2-branes, which are D4-branes wrapping the collapsed 2-cycle at the tip of the $\mathbb{C}^2/\mathbb{Z}_2$ ALE.

An important ingredient in understanding the gravity dual is the fact that there are at least three notions of charge which become distinct in the presence of fluxes. This issue was first emphasized by Marolf [23] who clarified the difference between brane, Page, and Maxwell charges. Brane charges are localized and gauge invariant but not quantized or conserved. Page charges are localized, quantized, and conserved but not gauge invariant. Maxwell charges are conserved and gauge invariant but not localized or quantized. In the context of field theories of the type under consideration, this issue was analyzed in detail in [6]. We will mostly follow the conventions of [6] and review these concepts as needed. We also refer the reader to appendix B of [6] where a set of useful formulas are collected. For the models depicted in figure 1a, N is the D2 Page charge and M is the D4 Page charge.

One can then think of the IIA solution as a dimensional reduction of M-theory on $R^{1,2} \times (\mathbb{C}^2/\mathbb{Z}_2) \times TN_k$ to which we add the back reaction of D2 and D4 branes sources. It is therefore natural to consider an ansatz where $R^{1,2} \times (\mathbb{C}^2/\mathbb{Z}_2) \times TN_k$ gets warped as a result of fluxes sourced by the D2 and the D4-branes.

The ansatz considered in [6] is

$$ds^2 = H^{-2/3}(-dt^2 + dx_1^2 + dx_2^2) + H^{1/3}(ds_{ALE}^2 + ds_{TN_k}^2), \quad (2.1)$$

$$G_4 = dC_3 = dt \wedge dx_1 \wedge dx_2 \wedge dH^{-1} + G_4^{\text{SD}}, \quad (2.2)$$

$$G_4^{\text{SD}} = d(lV\omega_2 \wedge \sigma_3 + 2\alpha\omega_2 \wedge d\psi) . \quad (2.3)$$

Our conventions are as follows:

- The 2-form w_2 is a self-dual 2-form on the collapsed cycle of the ALE normalized by:

$$\int_{ALE} w_2 \wedge w_2 = \frac{1}{2}. \quad (2.4)$$

- The M-Theory circle is parameterized by ψ living in the (unresolved) TN_k , whose metric is given by:

$$ds_{TN_k}^2 = V(r)^{-1} (dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)) + V(r) R_{11}^2 k^2 \left(d\psi - \frac{1}{2} \cos \theta d\phi \right)^2 \quad (2.5)$$

with

$$V(r) \equiv \left(1 + \frac{k R_{11}}{2r} \right)^{-1}, \quad R_{11} = l_s g_s. \quad (2.6)$$

- σ_3 is a one-form form living on TN_k defined as¹

$$\frac{1}{2} \sigma_3 = d\psi - \frac{1}{2} \cos(\theta) d\phi \quad (2.7)$$

where $\psi = \psi + 2\pi/k$.

- We may also introduce a parameter Q_2^{brane} in the sugra equation for the warp factor

$$0 = (\nabla_y^2 + \nabla_{TN}^2) H + \frac{l^2 V^4}{2r^4} \delta^4(\vec{y}) + (2\pi l_p)^6 Q_2^{\text{brane}} \delta^4(\vec{y}) \delta^4(\vec{r}). \quad (2.8)$$

Note that despite the fact that the parameter α in equation (2.3) is the coefficient of a total derivative term and would hence seem to be trivial, it turn out to be integrally quantized and related to the D4 Page charge via:

$$2\pi\alpha = (2\pi l_s)^3 g_s M. \quad (2.9)$$

Also, l is determined in terms of the Page charge, M , and b_∞ via:

$$l = -(2\pi l_s)^3 g_s \left(M + \frac{k b_\infty}{2} \right). \quad (2.10)$$

The only remaining ingredient is the brane charge Q_2^{brane} . This was found to be:

$$Q_2^{\text{brane}} = N - \frac{M^2}{k} = \frac{R_{AdS_4}^6}{64k\pi^2 l_p^6}. \quad (2.11)$$

When $Q_2^{\text{brane}} < 0$, the warp factor (as determined by equation (2.8)) becomes negative at sufficiently small radii giving rise to a repulson singularity. In general, for theories with 8 real susy generators, we expect that such singularities are always masked within an enhancon sphere at an even larger radius. This is indeed what was found in [7]. The enhancon radius is defined by the appearance of tensionless probes, which is equivalent to the divergence of the effective gauge coupling.

To study this issue, we may insert probe D4 branes wrapping the collapsed cycle of the *ALE*. One may label these probes by the D4 charge and dissolved D2 charge. One finds that these experience a Taub-NUT moduli space with NUT charges of $nk \mp 2M$, where

¹The subscript ‘3’ on this form comes from its role as the Cartan-Weyl one-form under the identification $SU(2) = S^3$.

$n = \mp \#D2$ and the upper/lower sign is taken for positive/negative D4 charge. Using the appropriate probes, one finds that the strength of the gauge couplings can be parameterized by a dimensionless parameter

$$\frac{1}{g_{\text{eff1}}^2} = b_\infty - \frac{g_{YM}^2 2M}{4\pi\Phi} \quad (2.12)$$

$$\frac{1}{g_{\text{eff2}}^2} = (1 - b_\infty) + \frac{g_{YM}^2 (k + 2M)}{4\pi\Phi} \quad (2.13)$$

where

$$g_{\text{eff1,2}} = \frac{g_{YM1,2}^2}{g_{YM}^2} \quad (2.14)$$

for gauge groups $U(N)$ and $U(N + M)$, respectively, with

$$\frac{1}{g_{YM}^2} = \frac{1}{g_{YM1}^2} + \frac{1}{g_{YM2}^2} = g_s/l_s \quad (2.15)$$

and

$$\Phi = \frac{1}{2\pi\alpha'} r \quad (2.16)$$

is the scale of renormalization being probed by the brane.

From the form of the probe action, it is straightforward to infer if and when an enhancon will arise from a probe whose (D2, D4) charge is $(n, \pm 1)$. This happens whenever $1/g_{\text{eff1}}^2$ or $1/g_{\text{eff2}}^2$ vanishes. It is clear from (2.12) and (2.13) that this will not happen provided (1.2) is satisfied. This is how one sees (1.2) arising on the supergravity side as was described in [7].

One natural generalization to this class of models is to include flavors charged under $U(N + M)$ as well as $U(N)$, as illustrated in figure 3. In [6], it was conjectured based on consideration of brane shuffling analysis that the gravity dual for these models should correspond to shifting the Page charge²

$$Q_2^{\text{Page}} = N + \frac{k_1}{4} \quad (2.17)$$

$$Q_4^{\text{Page}} = M - \frac{k_1}{2} . \quad (2.18)$$

However, a formal derivation of this relation from a purely supergravity consideration was not provided. We will show in the following sections that the conjecture of [6] is indeed correct.

3 Brane construction

The class of theories that we are interested in may be represented by Hanany-Witten brane constructions consisting of NS5, D5, and D3 branes with the following orientation:

	0	1	2	3	4	5	6	7	8	9
NS5	○	○	○	○	○	○				
D5	○	○	○					○	○	○
D3	○	○	○				○			

(3.1)

²See equation (2.82) of [6].

Arranging these elements as in figure 4, we may construct an $\mathcal{N} = 4$ theory in $2 + 1$ dimensions with gauge group of the form $\prod_{i=1}^{\hat{k}} \text{U}(N_i)$, bi-fundamental hypermultiplets charged under neighboring gauge groups, and k_i hypermultiplets charged under $\text{U}(N_i)$. The coordinates 3, 4, 5 labeling the transverse D5 position will be denoted by a set of k 3-vectors, \vec{x}_j . Likewise, the transverse positions of the \hat{k} NS5 branes will be denoted by \hat{k} vectors, \vec{y}_i , labeling the 7, 8, 9 coordinates.

In drawing figure 4 we have chosen the $\text{U}(N_{\hat{k}})$ interval as the one to “cut” open to present the circular quiver in a linear form. This allows us to define the concepts of “left” and “right” in a circular quiver. Opening the quiver at a node other than $\text{U}(N_{\hat{k}})$ will permute the notation but not otherwise affect the physics in any way.

In this diagram, \hat{b}_i labels the position of the i -th NS5 brane (denoted NS5 $_i$) before brane bending in the sense of [24] is considered. In other words, \hat{b}_i is the limit of the x^6 position of NS5 $_i$ as $|\vec{x}| \rightarrow \infty$. We will choose the indexing of the branes such the the sequence $(\hat{b}_1, \hat{b}_2, \dots, \hat{b}_{\hat{k}})$ is non-increasing as in figure 4 and the x^6 origin will be defined so that $\hat{b}_{\hat{k}} = 0$. This then gives rise to $\hat{k} - 1$ continuous parameters. The choice of ordering is made so as to guarantee that the asymptotic gauge couplings will be positive. The UV couplings may be read off from the HW diagram as:

$$\frac{1}{2g_{YM}^2} = \frac{L \left(\hat{b}_i - \hat{b}_{i+1} \right)}{g_s^{IIB}}. \quad (3.2)$$

We may also assign an asymptotic position, $b^j(\infty)$, to each D5 brane. This would increase the number of continuous parameters from $\hat{k} - 1$ to $\hat{k} + k - 1$. The position of b^k relative to $\hat{b}_{\hat{k}}$ is physically meaningful. We will see below that as one flows from defect field theory in 3+1 dimensions to the gauge theory in 2+1 dimensions, these k degrees of freedom parameterized by b^j decouples.

Resolving the *ALE* and the Taub-NUT corresponds to shifting the NS5 and the D5-branes in the transverse space, respectively. These, in turn, correspond to FI and mass parameters in the field theory, respectively. The relationship between the D5 position and the field theory mass parameter is easy to determine since the mass parameter is simply the mass of a fundamental string stretching between the D5 and a D3 stack. The scaling of the FI parameters may similarly be fixed in terms of the mass of a D-string as is illustrated in figure 5. We find:

$$\vec{m}_j = \frac{\vec{x}_j}{2\pi\alpha'} \quad (3.3)$$

$$\vec{\xi}_i = \frac{\vec{y}_i}{2\pi g_s^{IIB} \alpha'}. \quad (3.4)$$

For generic \vec{x}_j and \vec{y}_i , it is no longer possible for a D3 to end on a pair of D5 branes or NS5 branes while preserving supersymmetry. There are, however, supersymmetric configurations corresponding to D3 segments extending between a D5/NS5 pair as in figure 6. The brane configuration is thus specified by giving the number of D3 segments between each NS5 $_i$ and D5 $_j$ pair for the given choice of cut. For the sake of systematically displaying

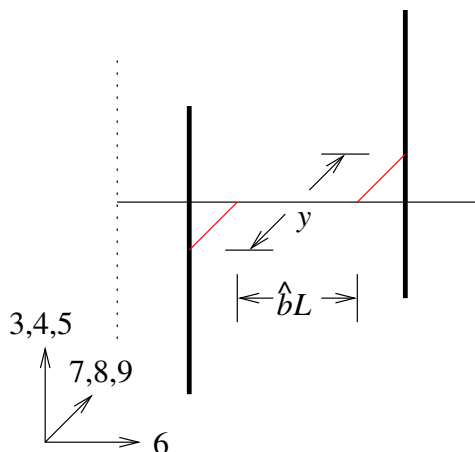


Figure 5. Determination of field theory FI parameter using BPS properties and s-duality. The black dots represent NS5 branes that are displaced in their transverse dimensions by a distance y . The periodicity of x_6 is taken to be L . The red lines represent D1 strings breaking on the D3 and their BPS mass determines the FI term.

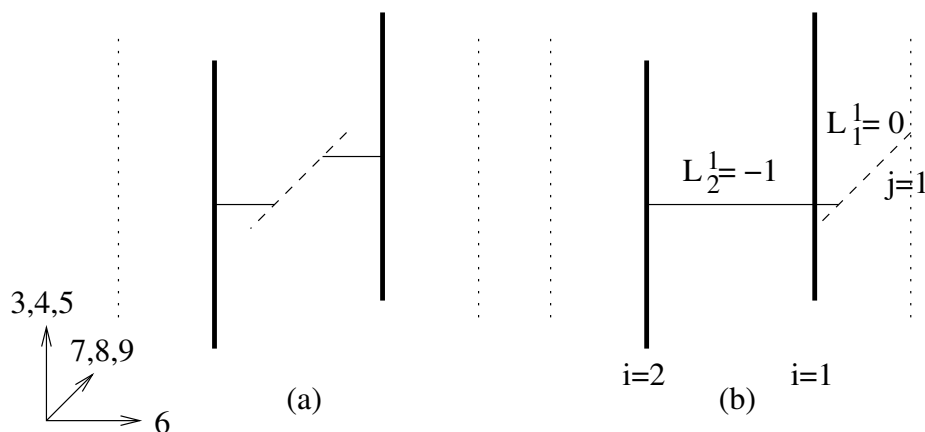


Figure 6. Hanany-Witten diagram (a) for a configuration allowing FI deformation without breaking any supersymmetries, and (b) the same configuration with the D5-branes pushed to right such that the data L_i^j is manifest.

this data, it is more convenient to move all the D5 branes to the right using Hanay-Witten transitions [25]. We will end up with a diagram such as figure 6. In figure 6b we illustrate a possible pattern of linkings corresponding to a resolved version of figure 6a. The parameters $L_i^j \in \mathbb{Z}$ characterize the allowed configurations and correspond to the number of D3 segments extending from the i -th NS5 to the j -th D5.

Note that on a linear quiver the s -rule implies that only a single D3 segment may connect a given NS5/D5 pair, hence implying L_i^j taking values,³ 0, 1, or -1 . This restriction is weaker on a cricular quiver since we can allow the D3 segment to wrap multiple times as in figure 7. This is equivalent to satisfying the s -rule in the covering space. For any

³The orientation is chosen such that negative L_i^j corresponds to a D3 segment ending on the left of the D5, while positive L_i^j corresponds to a D3 extending from the right.

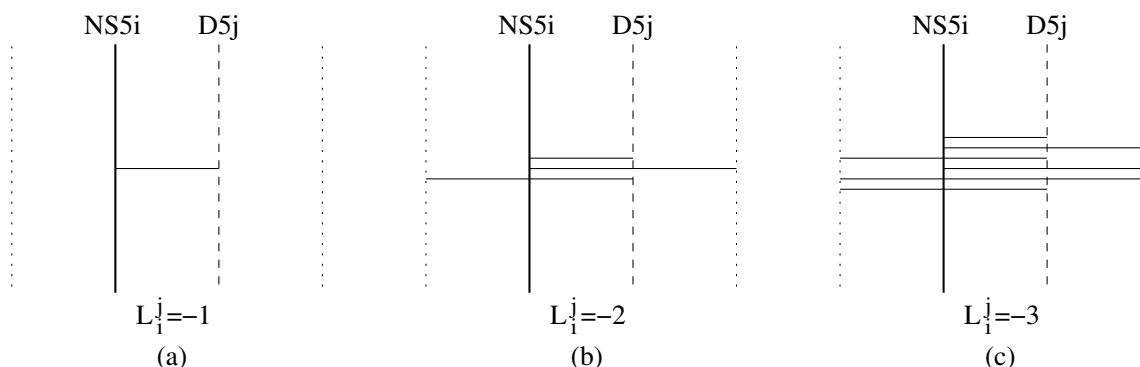


Figure 7. Multiple wrappings are allowed on a circular quiver.

given value of L_i^j , we will always take the brane configuration to be the one with the minimal number of wrappings consistent with the s -rule. For example, in figure 7, we must wrap the fractional brane, 0, 1, and 3 times for $L_i^j = -1, -2, -3$, respectively. We could consider introducing more than the minimal number of wrappings at each step; for example, we could have introduced 0, 2, and 5 wrappings instead for the same sequence of L_i^j 's. However, the freedom to do this is already encompassed by the freedom to add N_2^{free} integer branes. The integer N_2^{free} will turn out to be related to the number of M2-branes one needs to add to the resolved $ALE \times TN$ space. For now, we will analyze the configuration where N_2^{free} is set to zero, corresponding to a configuration where all of the Coulomb-branch is lifted. It will be straightforward to add these extra N_2^{free} branes at a later stage.

In figure 7, for instance, we see that for $L_i^j = -1, -2, -3 \dots$. We need to introduce at least $|L_i^j| - 1$ wrappings at the last step. Thus, the total number of wrappings associated with L_i^j that we *must* add is equal to:

$$\begin{aligned}
 \# \text{wrappings} &= 0 + 1 + 2 + \dots + (|L_i^j| - 1) \\
 &= \frac{|L_i^j|(|L_i^j| - 1)}{2} \\
 &= \frac{L_i^j (L_i^j + 1)}{2} .
 \end{aligned} \tag{3.5}$$

In the last step we used the fact that $L_i^j < 0$ in the example above. If L_i^j had been positive, similar logic would have yielded a total number of extra wrappings equal to $L_i^j(L_i^j - 1)/2$.

A useful intermediate concept is the linking numbers [25]. These are essentially monopole charges on the world-volumes of the 5-branes and their utility lies in the fact that they are invariant under Hanay-Witten brane maneuvers. Following, [26], we take the linking numbers to be defined as

$$\begin{aligned}
 l^j &= (\text{net D3's ending on left}) + (\text{NS5's to the right}) \\
 \hat{l}_i &= (\text{net D3's ending on right}) + (\text{D5's to the left}) .
 \end{aligned} \tag{3.6}$$

An inspection of the diagram shows that the linking numbers and the L_i^j are related by:

$$l^j = - \sum_{i=1}^{\hat{k}} L_i^j, \quad \hat{l}_i = - \sum_{j=1}^k L_i^j. \quad (3.7)$$

It should be clear that this map from the brane data L_i^j to linking numbers is many to one. Indeed, at this stage, L_i^j is an unconstrained $k \times \hat{k}$ array while \hat{l}_i and l^j are merely $\hat{k} + k$ variables with one constraint; $\sum_i \hat{l}_i = \sum_j l^j$. We therefore have $k\hat{k} - k - \hat{k} + 1$ extra pieces of information. This redundancy exist in the map from L_i^j to discrete field theory data, (N_i, k_i) , as is illustrated in figure 8. Some of the extra information corresponds to a choice of vacua within the specified theory.

However, not all configurations parameterized by distinct values of L_i^j are physically distinct when classifying the corresponding field theory vacua in $2 + 1$ dimensions. If we rotate the j -th D5 around the circle p^j times then the linking number of this particular D5 will change by $-\hat{k}p^j$ and the linking number of each NS5 will change by $-p^j$. It is straightforward to check that the full set of such transformations allows one the freedom to shift L_i^j (and hence \hat{l}_i, l^j) as

$$\begin{aligned} L_i^j &\rightarrow L_i^j + p^j \\ l^j &\rightarrow l^j - \hat{k}p^j \\ \hat{l}_i &\rightarrow \hat{l}_i - \sum_{j=1}^k p^j \end{aligned} \quad (3.8)$$

where p^j is an integer. We remark that the convention illustrated in figure 7 for defining the brane configuration in terms of L_i^j is invariant under this operation. This suggest that we may shift to

$$0 \leq l^j \leq \hat{k} - 1 \quad (3.9)$$

and obtain all physically distinct configurations. This is possible because the position of the D5 is irrelevant and has decoupled in flowing to the $2 + 1$ dimensional field theory. The condition (3.9) may be implemented by using (3.8) to replace the original L_i^j by

$$L_i^j \rightarrow L_i^j - \left\lceil \frac{1}{\hat{k}} \sum_{p=1}^{\hat{k}} L_p^j \right\rceil \quad (3.10)$$

where $\lceil \cdot \rceil$ is the ceiling function. In the following formulas we will assume that L_i^j already satisfies (3.9) and so suppress the transformation (3.10).

Now that we have enumerated the brane configurations (for $N_2^{\text{free}} = 0$) in terms of L_i^j and \hat{b}_i , it would be useful to understand how these data are related to the data N_i and k_i which appear in figure 4. It is in terms of the latter parameterization that the gauge and matter content of the field theory is easy to read off.

This can be acheived by basically inverting the process illustrated in figure 6 by moving the D5-branes to the right until there are no D3-branes ending on them. The condition (3.9)

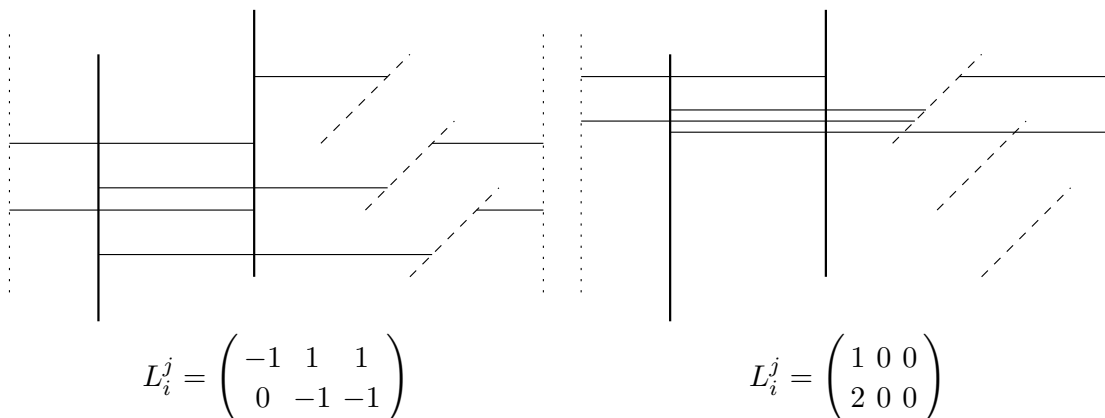


Figure 8. Different choices for L_i^j may give rise to the same field theory in the orbifold limit.

will guarantee that this procedure will terminate before the D5-branes circumnavigate the periodic x_6 direction. One can easily infer that this procedure leads to the relation

$$k_i = \#\{j | l_j = i \mod \hat{k}\} \quad (3.11)$$

$$\hat{l}_i = N_{i-1} - N_i + \#\{j | l_j \geq i\} . \quad (3.12)$$

The exercise was carried out in section 2.3 of [8]. Our (3.12) is identical to (2.5) of [8].

The relation (3.12) can be used to partially solve for N_i 's

$$N_i = \sum_{p=1}^i \sum_{j=1}^k L_p^j + \sum_{p=1}^i \#\{j | l_j \geq p\} + N_{\hat{k}} . \quad (3.13)$$

The relation (3.12) does not contain the information needed to fix $N_{\hat{k}}$, but that can also be extracted from the L_i^j data.

The issue is closely related to the counting of wrappings discussed previously in (3.5). It is easy to see the pattern by working out few simple cases with small values of L_i^j with some care in considering the cases with positive and negative values of L_i^j . Regardless, one can show that

$$N_{\hat{k}} = N_2^{\text{free}} + \frac{1}{2} \sum_{i=1}^{\hat{k}} \sum_{j=1}^k (L_i^j (L_i^j + 1)) \quad (3.14)$$

where for sake of completeness, we included the possibility to add arbitrary non-negative integer N_2^{free} corresponding to additional integer branes. The smallest allowed value of $N_{\hat{k}}$ then corresponds to setting N_2^{free} to zero, and that

$$N_{\hat{k}}^{\text{min}} = \frac{1}{2} \sum_{i=1}^{\hat{k}} \sum_{j=1}^k (L_i^j (L_i^j + 1)) \quad (3.15)$$

which can also be seen to be strictly non-negative provided L_i^j 's are integer valued. Since the choice of which of the N_i 's to identify as $N_{\hat{k}}$ was arbitrary, it also follows that all of the N_i 's must be positive definite.

3.1 Summary of brane construction

Since quite a bit of technical issues were discussed in this section, let us pause and summarize the main results. We enumerated the distinct brane configurations consisting of \hat{k} NS5-branes and k D5-branes in terms of the following data

L_i^j	Number of D3-branes stretching between NS5 _{<i>i</i>} and D5 ^{<i>j</i>}
N_2^{free}	Number of additional integer D3-branes
\vec{x}_j	Position of D5 ^{<i>j</i>} in $x_{3,4,5}$ coordinates
\vec{y}_i	Position of NS5 _{<i>i</i>} in $x_{7,8,9}$ coordinates
\hat{b}_i	Position of NS5 _{<i>i</i>} the periodic x_6 coordinate

with the following additional comments.

- i takes values in the range $1 \leq i \leq \hat{k}$ and j takes values in the range $1 \leq j \leq k$.
- L_i^j consists of $k \times \hat{k}$ integers subject to constraint (3.9) modulo permutation of j .
- N_2^{free} is a non-negative integer.
- \hat{b}_i gives rise to $\hat{k} - 1$ continuous parameters taking values in the range $0 = \hat{b}_k \leq \hat{b}_{\hat{k}-1} \leq \dots \leq \hat{b}_2 \leq \hat{b}_1 \leq 1$. \hat{b}_i is dimensionless. The physical position in the x_6 coordinate is given by $x_{6i} = L\hat{b}_i$ where L is the period of the x_6 coordinate.

These data were then mapped to

N_i	Rank of $U(N_i)$ gauge group in a circular quiver
k_i	Number of fundamtals charged with respect to $U(N_i)$
$\vec{\xi}_i$	Fayet-Illiopolous parameter for $U(N_i)$.
\vec{m}_j	Mass of the j -th fundamental charged with respect to $U(N_i)$ for $i = (l_j \mod \hat{k})$
$g_{YM i}^2$	Gauge coupling of $U(N_i)$

The map relating these to sets of data were presented in equations (3.2), (3.3), (3.4), (3.11), (3.13), and (3.14). For the convenience of the reader, they are also collected below.

$$\frac{1}{2g_{YM i}^2} = \frac{L(\hat{b}_i - \hat{b}_{i+1})}{g_s^{IIB}} \quad (3.16)$$

$$\vec{m}_j = \frac{\vec{x}_j}{2\pi\alpha'} \quad (3.17)$$

$$\vec{\xi}_i = \frac{\vec{y}_i}{2\pi g_s^{IIB} \alpha'} \quad (3.18)$$

$$k_i = \#\{j | l_j = i \mod \hat{k}\} \quad (3.19)$$

$$N_i = \sum_{p=1}^i \sum_{j=1}^k L_p^j + \sum_{p=1}^i \#\{j | l^j \geq p\} + N_{\hat{k}} \quad (3.20)$$

$$N_{\hat{k}} = N_2^{\text{free}} + \frac{1}{2} \sum_{i=1}^{\hat{k}} \sum_{j=1}^k \left((L_i^j)^2 + L_i^j \right) . \quad (3.21)$$

The discrete data are contained in L_i^j or the set (N_i, k_i) . As we noted previously, the mapping is not one to one. This reflects the fact that a field theory identified by (N_i, k_i) might admit more than one vacua. The L_i^j provides a parameterization which is distinct for each of these vacua.

It would be useful to develop some feel with regards to which values of (N_i, k_i) arise as corresponding to some L_i^j . This is equivalent to enumerating the set of (N_i, k_i) which admits a supersymmetric vacuum for a generic value of the FI and the quark mass parameters. Unfortunately, the relations (3.16)–(3.21) are a bit too cumbersome to convey that intuition in general, but one can easily analyze this issue explicitly for simple cases such as taking $\hat{k} = 2$. This is precisely the case where the quiver diagram takes the form illustrated in figure 3.

For this case, equations (3.7), (3.9), and (3.11) simplify to

$$l^j = -L_1^j - L_2^j = \begin{cases} 0 & \text{for } 1 \leq j \leq k_1 \\ 1 & \text{for } k_1 < j \leq k . \end{cases} \quad (3.22)$$

We may use this to eliminate L_2^j in the formulas (3.14) and (3.13) for N_1 and N_2 . Let us further restrict to the case where $k_2 = k$, corresponding to the quiver illustrated in figure 1a. Then, for $N_2 = N$ and $M = N_1 - N_2$, we find

$$N = N_2^{\text{free}} + \sum_{j=1}^k (L_2^j)^2 \quad (3.23)$$

$$M = - \sum_{j=1}^k L_2^j \quad (3.24)$$

for the allowed values of N_2^{free} and L_i^j . Figure 9 illustrates the set of N and M which can arise this way. As noted earlier, these correspond to the set of (N, M) for which a supersymmetric state exists for generic values of \vec{x}_j and \vec{y}_i . For now, let us simply note that

- The set of (N, M) closely resembles the parabola $Q_2^{\text{brane}} \geq 0$ for Q_2^{brane} given in (1.3) but misses some of the regions even in the large k limit.
- The set of (N, M) is contained in but does not saturate the region defined by (1.1).

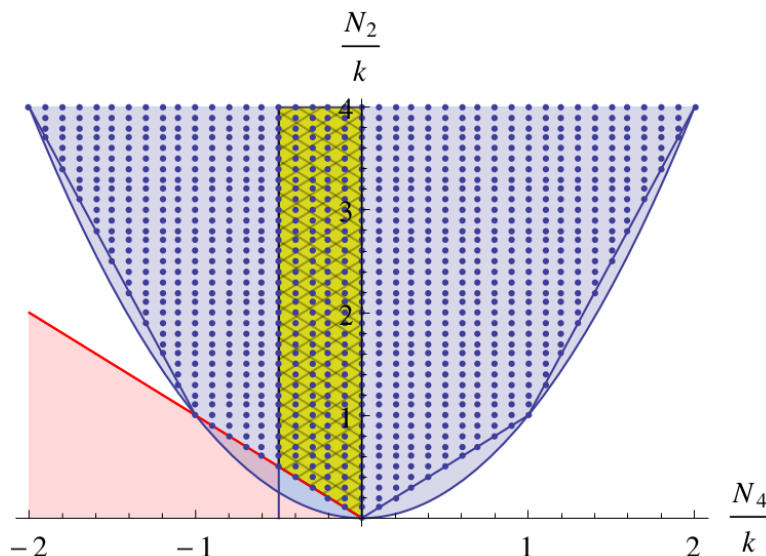


Figure 9. Dots represent set of resolvable theories for $\hat{k} = 2$, $k = 10$, $k_1 = 0$. The yellow region is the “good” region for which no enhancon appears.

4 Gravity dual description

In this section, we construct the gravity duals of brane configuration enumerated in the previous section. Just like in the examples considered in [6] and [7], start by applying T-duality along x_6 direction to map the system to IIA, and lift to M-theory. The \hat{k} NS5-branes and k D5-branes in the original IIB frame will map to a space whose structure is roughly that of $R^{1,2} \times TN_{\hat{k}} \times TN_k$ where the subscript denote the Taub-NUT charge. Since we are considering the setup where the \hat{k} FI parameters and k mass parameters are allowed to take generic values, we will treat these multi-centered Taub-NUT geometry is generically resolved.

Adding integer and fractional branes then corresponds to adding sources and letting the geometry warp. Just as was the case in earlier work [6, 7], one can write an explicit ansatz for the fields in 11 dimensional supergravity of the form

$$\begin{aligned} ds^2 &= H^{-2/3} (-dt^2 + dx_1^2 + dx_2^2) + H^{1/3} (ds_{TN_{\hat{k}}}^2 + ds_{TN_k}^2) \\ G_4 &= -dH^{-1} dt \wedge dx_1 \wedge dx_2 + G_4^{\text{SD}} \end{aligned} \quad (4.1)$$

and check that the BPS condition and subsequently the equation of motion is satisfied, provided that G_4^{SD} is a self-dual 4-form in $TN_{\hat{k}} \times TN_k$, and the warp factor H satisfies a Poisson equation

$$\nabla_{TN_{\hat{k}} \times TN_k}^2 H = -\frac{1}{2} *_8 (G_4^{\text{SD}} \wedge G_4^{\text{SD}}) - \sum_{m=1}^{Q_{M2}} (2\pi l_p)^6 \delta^8(\vec{z} - \vec{z}_m) \quad (4.2)$$

also on $TN_{\hat{k}} \times TN_k$. Here, \vec{z}_m denotes the position of an M2-brane in $TN_{\hat{k}} \times TN_k$.

Some properties of resolved Taub-NUT geometry will play an important role in the analysis below, so let us briefly recall the key facts.

A k -centered Taub-NUT is a four dimensional Euclidian geometry which can be viewed as an S^1 fibered over an R^3 . It has a metric

$$ds_{TN_k}^2 = V R_{11}^2 \left(d\psi - \frac{1}{2} \mathbf{w} \right)^2 + V^{-1} d\vec{x} \cdot d\vec{x} \quad (4.3)$$

where $V(\vec{x}, \psi)$ is a scalar function

$$V^{-1} = 1 + \sum_{j=0}^{k-1} \frac{R_{11}}{2|\vec{x} - \vec{x}_j|} \quad (4.4)$$

and

$$\mathbf{w} = \vec{\mathbf{w}} \cdot d\vec{x} \quad (4.5)$$

is a one form, and a vector $\vec{\mathbf{w}}$ is defined by

$$\frac{2}{R_{11}} \vec{\nabla} V^{-1} = \vec{\nabla} \times \vec{\mathbf{w}} . \quad (4.6)$$

The variable ψ must have a periodicity of 2π in order for the solution to be geometrically smooth at the Taub-NUT centers $\vec{x} = \vec{x}_j$ where the fiber along the ψ direction degenerates. Similar formulas hold for the TN_k factors as well, with $\tilde{L} \equiv \alpha'/L$ playing the role of R_{11} and the parameters \vec{x}_j being replaced by \vec{x}_i .

The positions of the Taub-NUT centers \vec{x}_i and \vec{x}_j will be interpreted as parameterizing the FI and the mass parameters, respectively, and are related to the positions of the NS5 and D5-branes as was given in (3.3) and (3.4). By relating the spectrum of BPS objects, they can be mapped to the following relation in M-theory.

$$\vec{m}_j = \frac{R_{11} \vec{x}_j}{2\pi l_p^3} \quad (4.7)$$

$$\vec{\xi}_i = \frac{\tilde{L} \vec{x}_i}{2\pi l_p^3} . \quad (4.8)$$

The k -centered Taub-NUT geometry has k normalizable anti-self-dual 2 forms of which $k - 1$ is Poincare dual to compact 2-cycles. They can be parameterized as

$$w_j = d\lambda_j \quad (4.9)$$

with the one form λ_j given by

$$\lambda_j = \frac{1}{4\pi} \left(2f_j \left(d\psi - \frac{1}{2} \mathbf{w} \right) - \sigma_j \right) \quad (4.10)$$

where

$$\sigma_j = \cos \theta_j d\phi_j \quad (4.11)$$

is the potential for the volume form of a unit sphere⁴

$$d\Omega_j = d\sigma_j \quad (4.12)$$

centered at \vec{x}_j in R^3 , and we have defined

$$f_j = -\frac{R_{11}}{2|\vec{x} - \vec{x}_j| \left(1 + \sum_{\sigma} \frac{R_{11}}{2|\vec{x} - \vec{x}_{\sigma}|}\right)}, \quad (4.13)$$

and the 1-form \mathbf{w} is written in terms of σ_j 's as

$$\mathbf{w} \equiv \sum_j \sigma_j. \quad (4.14)$$

These self-dual two forms are normalized so that

$$\int_{TN_k} w_m \wedge w_j = \delta_{mj}. \quad (4.15)$$

It is straight forward then to parameterize the self-dual 4-forms on $TN_{\hat{k}} \times TN_k$ as

$$G_4 = (2\pi l_p)^3 M_i^j \hat{w}^i \wedge w_j \quad (4.16)$$

where M_i^j is a dimensionless numerical coefficient. There are $\hat{k} \times k$ independent components in M_i^j corresponding to \hat{k} and k linearly independent 2-forms on $TN_{\hat{k}}$ and TN_k , but they are subject to flux quantization and boundary conditions. In order to understand these constraints, it is useful to visualize the 2-cycles on multi-centered Taub-NUT geometry which we illustrate in figure 10.

We begin by noting that there exists a collection of 2-cycles which we denote l_i which is a semi-infinite line on R^3 base of Taub-NUT geometry with S^1 fiber over each point. This cycle is smooth at each of the Taub-NUT centers labeled as x_i . The period of the two forms w_j on the l_i cycles takes a simple form

$$\int_{l_i} w_j = \delta_{ij}. \quad (4.17)$$

An alternative scheme to identify the homology cycles is to define $k - 1$ cycles

$$\Sigma_i = l_{i+1} - l_i, \quad i = 1 \dots k - 1 \quad (4.18)$$

as is illustrated in figure 10b. These cycles are compact. Similarly, there are $\hat{k} - 1$ cycles $\hat{\Sigma}_i$ on $TN_{\hat{k}}$. There are therefore $(\hat{k} - 1) \times (k - 1)$ compact 4 cycles in $TN_{\hat{k}} \times TN_k$.

Clearly, the period of G_4 on $\hat{\Sigma}_i \times \Sigma_j$ must be quantized. That is

$$P_i^j = \frac{1}{(2\pi l_p)^3} \int_{\hat{\Sigma}_i \times \Sigma_j} G_4^{\text{SD}} = \mathbb{Z}. \quad (4.19)$$

⁴Of course this expression is only valid in certain coordinate patches.

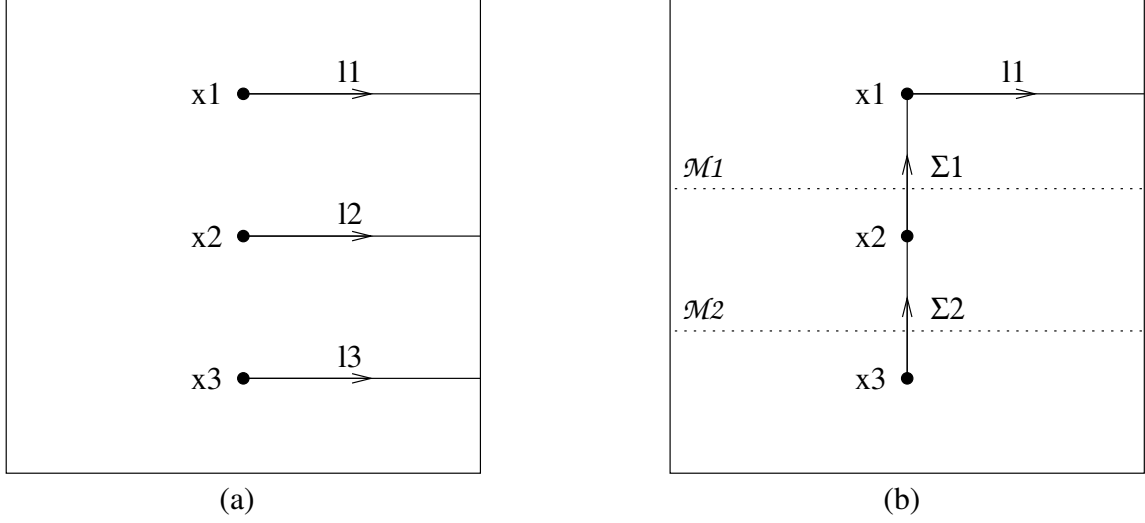


Figure 10. In figure (a) the l_i are semi-infinite lines in \mathbb{R}^3 with an S^1 fiber over each point. These are the generators of the non-compact homology. The compact homology, figure (b), is formed by fibering the ψ circle over lines between the Taub-NUT centers. These are homologically equivalent to the differences $l_{i+1} - l_i$. A similar figure appears in [27].

This constrains $(\hat{k} - 1) \times (k - 1)$ components of M_{ij} . The remaining $\hat{k} + k - 1$ components can only be characterized in a gauge invariant manner as a period over a non-compact cycle. Without loss of generality, we can take that cycle to be l_1 . Let us define

$$\hat{q}_i = \frac{1}{(2\pi l_p)^3} \int_{\hat{\Sigma}_i \times l_1} G_4^{\text{SD}}, \quad i = 1 \dots \hat{k} - 1 \quad (4.20)$$

$$q^j = \frac{1}{(2\pi l_p)^3} \int_{\hat{l}_1 \times \Sigma_j} G_4^{\text{SD}}, \quad j = 1 \dots k - 1 \quad (4.21)$$

$$r = \frac{1}{(2\pi l_p)^3} \int_{\hat{l}_1 \times l_1} G_4^{\text{SD}}. \quad (4.22)$$

The set of parameters $(P_i^j, \hat{q}_i, q^j, r)$ completely specify all components of M_{ij} .

$$M_i^j = r + \sum_{i'=1}^{i-1} \hat{q}_{i'} + \sum_{j'=1}^{j-1} q^{j'} + \sum_{i'=1}^{i-1} \sum_{j'=1}^{j-1} P_{i'}^{j'}. \quad (4.23)$$

To the extent that \hat{l}_1 and l_1 are non-compact, there is no sense in which \hat{q}_i , q^j , or r are to be quantized. There is, however, a meaning which can be attributed to the fractional part of \hat{q}_i , q^j , and r . The pullback of the 3-form potential

$$\hat{\gamma}_i = \frac{1}{(2\pi l_p)^3} \int_{\hat{\Sigma}_i \times \partial l_1} C_3^{\text{SD}} \simeq \hat{q}_i \mod \mathbb{Z} \quad (4.24)$$

$$\gamma^j = \frac{1}{(2\pi l_p)^3} \int_{\partial \hat{l}_1 \times \Sigma_j} C_3^{\text{SD}} \simeq q_j \mod \mathbb{Z} \quad (4.25)$$

$$\gamma = \frac{1}{(2\pi l_p)^3} \int_{\partial \hat{l}_1 \times l_1} C_3^{\text{SD}} \simeq r \mod \mathbb{Z} \quad (4.26)$$

has the same fractional parts as \hat{q}_i , q^j , and r up to large gauge transformation which can shift their values by integer amounts.⁵

It is convenient to define

$$\hat{b}_i = \sum_{i'=1}^{i-1} \hat{\gamma}_{i'}, \quad i = 1 \dots \hat{k} \quad (4.27)$$

$$b^j = \sum_{j'=1}^{j-1} \gamma_{j'}, \quad i = 1 \dots k \quad (4.28)$$

which can be arranged, after large gauge transformation and permutations, to satisfy

$$0 = \hat{b}_{\hat{k}} \leq \hat{b}_{\hat{k}-1} \leq \dots \leq \hat{b}_2 \leq \hat{b}_1 \leq 1 \quad 0 \leq \gamma \leq 1. \quad (4.29)$$

As is evident from the notation, \hat{b}_i (b^j) will be identified with the $NS5$ ($D5$) brane position in, for instance, equation (3.2). Also, note that we will not need to impose any condition on b_j . In this setup, we can write

$$M_i^j = L_i^j + \hat{b}_i + b^j + \gamma \quad (4.30)$$

where L_i^j consists of $\hat{k} \times k$ integer data and $\hat{k} + k - 1$ continuous data \hat{b}_i , b^j , and γ consisting of real numbers between 0 and 1, respecting the constraint (4.29). Roughly speaking, these data can be thought of as arising from separating the \hat{q}_i , q^j , and r into their integer part and the fractional parts. Notice, however, that at this point there is an ambiguity in specifying L_i^j since the large gauge transform (3.8) mixes L_i^j and b^j while keeping M_i^j invariant.

Fortunately, the ambiguity in L_i^j is rendered harmless in the limit that $TN_{\hat{k}} \times TN_k$ degenerates to $ALE_{\hat{k}} \times TN_k$. The key difference between $TN_{\hat{k}}$ and $ALE_{\hat{k}}$ is the fact that one linear combination of the \hat{k} self-dual 2-forms

$$\hat{\Xi} = \sum_{i=1}^{\hat{k}} \hat{w}^i \rightarrow 0 \quad (4.31)$$

no longer exists as an element of cohomology. This implies that G_4^{SD} is independent of γ and b^j . Hence, the physics is unaffected by $L_i^j \rightarrow L_i^j + p^j$, even for fixed arbitrary b^j . In other words, the counting of parameters is reduced to L_i^j having $\hat{k} \times k - k$ distinct discrete parameters and M_i^j having $\hat{k} - 1$ additional continuous parameters \hat{b}_i . The ambiguity/redundancy in L_i^j can be thought of as the manifestation of (3.8) we encountered in the previous section. The relations (3.16)–(3.21) were derived for L_i^j with ambiguity (3.8) fixed to satisfy (3.9). Since M_i^j decouples from γ and b^j , we can freely adjust L_i^j to have the same structure as what we found in the previous section. This provides compelling reason to identify the L_i^j introduced in equation (4.30) with the L_i^j appearing in the brane construction of the previous section. We therefore propose equation (4.30), with L_i^j having the aforementioned interpretation in terms of brane data, as the key element of the holographic dictionary for this class of theories. In the following section, we will provide further support for this identification by relating these discrete parameters to Page charges in type IIA supergravity.

⁵It should be noted that roughly speaking M corresponds to l and γ corresponds to $l - \alpha$ in the parameterization of (2.3).

5 IIA description

In the previous section, we considered a large class of BPS supergravity backgrounds for M-theory on $TN_{\hat{k}} \times TN_k$ with quantized fluxes as well as their $ALE_{\hat{k}} \times TN_k$ limit. In this section, we will consider the type IIA reduction of these backgrounds and relate the parameters to type IIB brane construction parameters considered earlier.

The IIA reduction of the ansatz (4.1) can be written in the form

$$\begin{aligned}
 A_3 &= -H^{-1} dt \wedge dx_1 \wedge dx_2 + (2\pi l_p)^3 M_{ij} \hat{w}_i \wedge \lambda_j \\
 B_2 &= -\frac{1}{2\pi R_{11}} (2\pi l_p)^3 M_{ij} \hat{w}_i f_j - \frac{2}{R_{11}} (2\pi l_p)^3 \hat{\beta}_i \hat{w}_i \\
 e^\phi &= H^{1/4} V^{-3/4} \\
 A_1 &= -\frac{R_{11}}{2} \sum_j \sigma_j
 \end{aligned} \tag{5.1}$$

where we have written the potential for the IIA form fields explicitly to match the boundary condition (4.24). We are also working in the $ALE_{\hat{k}} \times TN_k$ limit where some of the components of M_{ij} have decoupled. In the IIA descriptions, the branes are oriented as follows:

	0	1	2	3	4	5	6	7	8	9
NS5	○	○	○	○	○	○				
D6	○	○	○				○	○	○	○
D2	○	○	○							

(5.2)

One way in which we can explicitly relate the L_i^j which appeared in the context of gravity solution in the previous section to the L_i^j from the brane construction is to examine the magnetic flux on k D6-brane extended along the $ALE_{\hat{k}}$ parameterized by the (6789) coordinates, and localized along the 345 coordinates in the dimensional reduction of M-theory to IIA. The presence of a D3 segment stretched between the i -th NS5-brane and the j -th D5 brane in the IIB picture translates to the presence of magnetic flux on the D6 world volume which can be read off from the supergravity solution.

First, let us recall that the gauge invariant U(1) world volume gauge field on the j -th D6 brane can be read off using formulas in [28, 29] as follows:

$$\mathcal{F}^j = B + 2\pi\alpha' F^j = -\frac{1}{2\pi R_{11}} \int_{l_j} G_4^{\text{SD}} = -(2\pi l_s)^2 M_i^j \hat{\omega}_i. \tag{5.3}$$

Here, B is the induced NSNS 2-form on the D6-brane world volume in the probe approximation. That is, it should be the B -field evaluated at \vec{x} at infinity. From (5.1) we can read off

$$B = -(2\pi l_s)^2 \sum_i \hat{b}_i \hat{w}_i. \tag{5.4}$$

Combining these expressions, we find that

$$F^j = -2\pi \sum_i (M_i^j - \hat{b}_i) \hat{\omega}_i = -2\pi \sum_i L_i^j \hat{\omega}_i. \tag{5.5}$$

When this field strength is dimensionally reduced on $\hat{\psi}$, we obtain a U(1) gauge field on an R^3 with \hat{k} marked points indexed by i on the R^3 base space of $ALE_{\hat{k}}$. The world volume gauge field becomes that of L_i^j units of magnetic charge localized at the i -th point on the j -th D6-brane. This is precisely what one expects from the T-dual of a D3-brane stretched between the i -th NS5-brane and the j -th D5-brane in the type IIB picture. In the $ALE_{\hat{k}} \times TN_k$ limit of $TN_{\hat{k}} \times TN_k$, the precise value for the “center of mass” $\sum_i L_i^j$ to assign is ambiguous, corresponding to the physical ambiguity discussed in the context of brane configuration in (3.8). This ambiguity manifests itself as ambiguity with respect to large gauge transformation on the supergravity side.

Another quantity that is interesting to compute in the IIA description to clarify the mapping of parameters under gauge gravity correspondence is the Page charge. Page charge is one of three distinct notion of charges identified by Marolf [23] in background with fluxes and has the property of being localized, conserved, but not invariant under large gauge transformations in some cases. They are defined as periods of Page flux over cycles containing the charge source. A convenient reference for the Page fluxes and various subtle supergravity conventions we follow can be found in appendix B of [6].

The D6 Page charge is simply

$$Q_6^{\text{Page}} = \frac{1}{2\pi R_{11}} \int F_2 = k \quad (5.6)$$

and counts the number of D6-branes.

Similarly, we define the D4-Page charge associated to D4-branes warped on each of the $(\hat{k} - 1)$ 2-cycles $\hat{\Sigma}_i$ in $ALE_{\hat{k}}$ by integrating the D4 Page flux over the cycle $\hat{\mathcal{M}}_i$ orthogonal to $\hat{\Sigma}_i$ as was defined in figure 10b.

$$Q_{4i}^{\text{Page}} = \frac{1}{(2\pi l_p)^3} \int_{\hat{\mathcal{M}}_i \times S^2} (-\tilde{F}_4 - B_2 \wedge F_2) = -\frac{1}{(2\pi l_p)^3} \int_{\hat{\mathcal{M}}_i \times S^2} d(A_3 + B_2 \wedge A_1) \quad (5.7)$$

where S_2 is the sphere on R^3 containing all the centers of the TN_k viewed as an S^1 fibration over R^3 . Substituting the type IIA background, we find that these D4 Page charges evaluates to

$$Q_{4i}^{\text{Page}} = \frac{1}{2} \left(\sum_{n=i+1}^{\hat{k}} \hat{l}_n - \sum_{n=1}^i \hat{l}_n \right) + \frac{1}{2\hat{k}} \left(2i - \hat{k} \right) \sum_{n=1}^{\hat{k}-1} n k_n, \quad i = 1 \dots (\hat{k} - 1). \quad (5.8)$$

In performing this calculation, the following formula for computing the intersection form on $ALE_{\hat{k}}$ is useful.

$$\int_{ALE_{\hat{k}}} \hat{w}_i \wedge \hat{w}_j = \delta_{ij} - \frac{1}{\hat{k}}. \quad (5.9)$$

Also, it should be noted that Q_{4i}^{Page} is only defined modulo k due to gauge ambiguities.

Similar computation of the D2 Page charge gives

$$Q_2^{\text{Page}} = Q_{M2} + \frac{1}{2} \sum_{i,j} \left(L_i^j - \frac{1}{\hat{k}} \sum_i L_i^j \right)^2. \quad (5.10)$$

The fact that Q_2^{Page} and Q_{4i}^{Page} are all expressed in terms of discrete quantities is consistent with the expectation that the Page charges are discrete, conserved quantities. It may come as a bit of a surprise that these quantities do not take integer values themselves. Page charges, however, have been known to contain anomalous additive contributions, such as the Freed-Witten anomaly, that can shift their values away from strictly integral values. It was in fact anticipated that the Q_2^{Page} and Q_{4i}^{Page} takes on a specific form (2.17) and (2.18) for the model whose quiver diagram is illustrated in figure 3 with $\hat{k} = 2$. Using the data for L_i^j which we worked out in (3.22), we find that (2.17) and (2.18) are reproduced precisely. Although we have computed the total Page charge for the resolved theory, we expect the same result for the orbifold limit since Page charges, being discrete, are invariant under continuous deformations.

Another useful quantity that we can compute to characterize the type IIA background is the Maxwell charge, which is gauge invariant and conserved but is not invariant under deformation or quantized. It is given by

$$Q_2^{\text{Maxwell}} = \frac{1}{(2\pi l_s)^5 g_s} \int_{ALE_{\hat{k}} \times S^2} * \tilde{F} \quad (5.11)$$

and evaluates to

$$Q_2^{\text{Maxwell}} = Q_{M2} + \frac{1}{2} \sum_{i,j} \left(\hat{L}_i^j - \hat{\beta}_i - \frac{1}{\hat{k}} \sum_m (L_m^j - \hat{\beta}_m) \right)^2 \quad (5.12)$$

which also be written in the form

$$\begin{aligned} Q_2^{\text{Maxwell}} &= Q_2^{\text{Page}} + Q_{4m}^{\text{Page}} \int_{\hat{\Sigma}_m} B + \frac{k}{2} \int_{ALE_{\hat{k}}} B \wedge B \\ &= Q_2^{\text{Page}} + Q_{4i}^{\text{Page}} \hat{\gamma}_i + \frac{k}{2} (\mathcal{I}^{-1})_{ii'} \hat{\gamma}_i \hat{\gamma}_{i'} \end{aligned} \quad (5.13)$$

where $\mathcal{I}_{ii'}$ is the $(\hat{k} - 1) \times (\hat{k} - 1)$ intersection matrix (5.14) on $ALE_{\hat{k}}$:

$$\mathcal{I}_{ij} = \begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & -2 & -1 \\ & & & -1 & 2 \end{pmatrix}. \quad (5.14)$$

It is easy to interpret the Maxwell charge as the net D2 charge induced by the B -field being pulled back on the Q_{4i}^{Page} D4 and k D6 branes when written in this form. In this sense, the Page charges are counting the effective number of branes that are present albeit with some shift.

In the $\hat{k} = 2$ limit, this reduces to

$$Q_2^{\text{Maxwell}} = Q_2^{\text{Page}} + Q_4^{\text{Page}} \gamma_1 + \frac{k}{4} \gamma_1^2 \quad (5.15)$$

and agrees with the results previously reported in [6, 7] when γ_1 is identified as b_∞ .

Another important quantity which is useful for understanding the supergravity dual is the D2 brane charge and the bulk charge. The bulk charge refers to the contribution to the Maxwell charge arising from the $G_4 \wedge G_4$ term in the M-theory equation

$$d * G_4 - \frac{1}{2} G_4 \wedge G_4 = (2\pi l_p)^6 * j_{M2}^{\text{brane}} \quad (5.16)$$

in M-theory and the IIA reduction thereof.

So

$$Q_2^{\text{Bulk}} = \frac{1}{2(2\pi l_p)^6} \int_{ALE_{\hat{k}} \times TN_k} G_4 \wedge G_4 = \frac{1}{2} \sum_{i,j} \left(\hat{L}_i^j - \hat{\beta}_i - \frac{1}{\hat{k}} \sum_m (L_m^j - \hat{\beta}_m) \right)^2. \quad (5.17)$$

The brane charge is the contribution from the source term and simply evaluates to

$$Q_2^{\text{brane}} = \int_{ALE_{\hat{k}} \times TN_k} * j_{M2}^{\text{brane}} = Q_{M2}. \quad (5.18)$$

Here, we have formulate these quantities in the language of M-theory but they reduce naturally to type IIA and evaluates to the same value. This analysis was for the case where the $ALE_{\hat{k}}$ and TN_k were completely resolved and as such, the only possible localized source contributing to $*j_{M2}$ is the isolated M2/D2 brane. $Q_2^{\text{brane}} = Q_{M2}$ is therefore naturally an integer valued quantity.

That Q_2^{brane} is strictly integral and positive definite is curiously at odds with the earlier claim (1.3). It turns out that this apparent mismatch has some very interesting physical origin.

First, let us note that the self-dual 4-form can be decomposed into compact and non-compact part

$$G_4 = G_4^{\text{compact}} + G_4^{\text{non-compact}} \quad (5.19)$$

where

$$G_4^{\text{non-compact}} = \frac{(2\pi l_p)^3}{k} \left(\sum_{j=1}^k M_i^j \right) \hat{w}^i \wedge \left(\sum_j w_j \right) \quad (5.20)$$

and so

$$G_4^{\text{compact}} = G_4 - G_4^{\text{non-compact}} \quad (5.21)$$

$$\equiv (2\pi l_p)^3 \sum_{ij} \hat{M}_i^j \quad (5.22)$$

where G_4^{compact} is the part which has non-trivial period on Σ_j 's whereas

$$\int_{\Sigma_j} G_4^{\text{non-compact}} = 0. \quad (5.23)$$

The compact and non-compact components are orthogonal, so that

$$\frac{1}{2(2\pi l_p)^3} \int_{ALE_{\hat{k}} \times TN_k} G_4 \wedge G_4 = Q_2^{\text{compact}} + Q_2^{\text{non-compact}} \quad (5.24)$$

with

$$Q_2^{\text{compact}} = \frac{1}{2(2\pi l_p)^3} \int_{ALE_{\hat{k}} \times TN_k} G_4^{\text{compact}} \wedge G_4^{\text{compact}} \quad (5.25)$$

$$Q_2^{\text{non-compact}} = \frac{1}{2(2\pi l_p)^3} \int_{ALE_{\hat{k}} \times TN_k} G_4^{\text{non-compact}} \wedge G_4^{\text{non-compact}}. \quad (5.26)$$

The point of decomposing G_4^{SD} into compact and non-compact components is that they behave very differently in the orbifold limit of TN_k . The non-compact component has a smooth limit. Nothing special happens to the non-compact component in taking the orbifold limit.

Not too surprisingly, the compact component of the bulk charge exhibits more intricate behavior in the orbifold limit. The compact component of $G_4 \wedge G_4$, in fact, degenerates to a delta function in the orbifold limit. One way to see this explicitly in a simple example is to look at the compact self-dual 2-form on Eguchi-Hanson space in the orbifold limit given, for instance, in equation (B.4) of [30].

Upon explicit evaluation, one finds

$$Q_2^{\text{compact}} = \frac{1}{2} \left(\sum_{i,j} (\hat{M}_i^j)^2 - \frac{1}{\hat{k}} \sum_i \sum_{n,m} \hat{M}_n^j \hat{M}_m^j \right) \quad (5.27)$$

$$Q_2^{\text{non-compact}} = \frac{1}{2k} \left(\sum_{i,j,k} M_i^j M_i^k - \frac{1}{\hat{k}} \left(\sum_{i,j} M_i^j \right)^2 \right). \quad (5.28)$$

To the extent that Q_2^{compact} is delta-function supported at the tip of the orbifold, it behaves in many ways like a brane charge. Suppose for the sake of argument we consider the combination

$$Q_2^{\text{brane}} + Q_2^{\text{compact}} = Q_{M2} + \frac{1}{2} \sum_{i,j} \left(L_i^j + l^j / \hat{k} - \frac{1}{k} \sum_j \left(L_i^j + l^j / \hat{k} \right) \right)^2. \quad (5.29)$$

Now, using (5.10), one finds

$$Q_2^{\text{brane}} + Q_2^{\text{compact}} = Q_2^{\text{Page}} - \frac{1}{2k} \sum_i \left(\sum_j \left(L_i^j + \frac{l^j}{\hat{k}} \right) \right)^2 \quad (5.30)$$

which looks somewhat complicated. However, when restricted to $\hat{k} = 2$ and $k_2 = k$, which was the case considered in [6], this equation simplifies to

$$Q_2^{\text{brane}} + Q_2^{\text{compact}} = Q_2^{\text{Page}} - \frac{(Q_4^{\text{Page}})^2}{k} \quad (5.31)$$

which now can be seen as having the same form in the right hand side as (1.3).

We have therefore gained a useful perspective on why (5.18) and (1.3) gave seemingly different results. The two main difference is that 1) the earlier expression (1.3) included contribution from G_4^{compact} , and 2) was expressed in terms of Q_2^{Page} which differ from Q_{M2} as is given in (5.10).

In fact, when the $ALE_{\hat{k}} \times TN_k$ is resolved, (5.10) implies that

$$Q_{M2} = Q_2^{\text{Page}} - \frac{1}{2} \sum_{i,j} \left(L_i^j - \frac{1}{\hat{k}} \sum_i L_i^j \right)^2 > 0 \quad (5.32)$$

gives rise to a *stronger* condition for perserving supersymmetry than the condition that (1.3) be positive.

The only remaining issue is whether the bound (5.32) can be violated if large localized charge is present due to fluxes threading the collapsed cycles in the orbifold limit to ensure in (5.31) is positive. This is a subtle issue of topology change and is beyond the scope of classical gravity analysis. It would be very interesting to understand the local behavior of this system around this transition point, as this issue lies at the hart of how the meeting of Higgs and the Coulomb branch is captured in gauge/gravity duality.

6 Conclusions

In this article, we described an explicit construction of gravity dual of $\mathcal{N} = 4 \prod_{i=1}^{\hat{k}} U(N_i)$ quiver gauge theory with k_i fundamentals charged under $U(N_i)$ and bi-fundamentals, generalizing the earlier construction of [6]. The supergravity description we find captures the full renormalization group flow starting from the UV point in 2+1 dimensions, at least in the regime where the gravity approximation is reliable. Our construction consisted primarily of generalizing the structure of $ALE_2 \times TN_k$ used as the starting point in [6] to $ALE_{\hat{k}} \times TN_k$. We further resolved the $ALE_{\hat{k}}$ and TN_k geometry to be completely regular, and allowed fluxes to thread through the resulting compact four cycles. These fluxes turns out to encode the discrete data characterizing N_i , k_i , and the choice of vacua among the set of discrete, degenerate, choices. The mapping between the fluxes and these discrete data is somewhat cumbersome, but is accessible. The results are summarized in (3.16)–(3.21).

The quantization of fluxes and charges takes a relatively simple form when formulated as an exercise in M-theory on $R^{1,2} \times ALE_{\hat{k}} \times TN_k$ when the orbifold singularity is completely resolved. All the localized M2 sources corresponds to physical M2-branes, and as such have positive definite, integer quantized values for supersymmetric solutions. This is somewhat at odds with the previous finding of type IIA brane charge (2.11) whose values were only quantized in units of $1/k$. We found a gratifying resolution to this apparent discrepancy. Upon taking the orbifold limit, the fluxes threading the compact cycles approach an effective delta-function source at the orbifold fixed point. These sources can carry charges that are quantized in units of $1/k$ relative to the M2 charge. In essence, the flux thourgh the compact cycle transmutes to discrete torsion like in [31, 32].

What is interesting about the orbifold limit of our construction based on resolved $ALE_{\hat{k}} \times TN_k$ is that it should correspond to the point on the moduli-space where the

Coulomb branch and the Higgs branch meet. It would be very interesting to understand this transition concretely from the bulk description and account in detail features such as the dimension of Coulomb branch.

We seem to be finding, however, that much of the interesting field theory dynamics requires understanding the stringy resolution of classical gravity. For models whose parameters are outside the range (1.2), one must resolve the enhancons in order to extract meaningful physics. Even for models whose parameters are contained in the (1.2), string corrections become important when approaching the threshold of supersymmetry. In fact, the discrepancy between (1.1) and (1.3) can be attributed entirely to the stringy uncertainty as was outlined in the introduction. Similar stringy corrections also arise in ABJM model as was shown in [33]. If the nature of stringy corrections can be classified on the gravity side, perhaps one can use gauge gravity correspondence to also classify the stringy effects using known exact results such as the ones obtained via localization [13, 14].

Perhaps one of the most profound implication of the string correction is the status of gravity solution for parameters corresponding the edge of the parabola illustrated in figure 9. For solutions in the interior of the parabola in the enhancon free region (1.2), one expects an AdS_4 throat but at the edge of the parabola, that throat disappears and one seemingly obtains a regular geometry capped off near the origin. A large class of “regular” solutions of this type were constructed in a impressive body of work by Cvetič, Gibbons, Lu, and Pope starting with [34] and reviewed in [35]. The singularity free regular solution being constructed in these works primarily involved sitting at the edge of the parabola. Attempts to explore dynamical supersymmetry breaking along the lines of [19, 20] consisted of exploring the region near the edge of the parabola. The picture emerging from the consideration of string corrections, however, is the fact that the concept of tuning of parameter to sit at the edge of the parabola itself is subject to correction. This is because as the edge of the parabola is approached from the inside, the AdS_4 throat becomes highly curved, and the semi-classical formula relating the radius to charges starts to receive corrections. The issue here stems largely from if/whether one achieves the smooth, capped off geometry in the limit of vanishing AdS_4 radius. Recently, in a very interesting series of papers, it was argued that a tip of the $SL(2)/U(1)$ cigar receives α' corrections which are non-perturbative dramatically altering the semi-classical intuition that the tip of the cigar is smooth [36–38]. One way to understand the physics of stringy corrections in their context was to say that translation invariance in the winding space was broken by condensation of winding modes. Something very similar could be happening near the edge of the parabola as is illustrated in figure 11. There are indeed compelling evidence that translation invariance in the periodic direction T-dual to $\hat{\psi}$ is broken in the explicit construction of supergravity duals of the superconformal IR fixed point for theories in the “good” region (1.2) [26, 39]. Microscopic corrections to gravity that are not immediately apparent from the gravity as a low-energy effective field theory is at the heart of the black hole information paradox, and it is quite interesting to find a manifestation of such effects even in a highly supersymmetric setup like the ones considered in this note. These issues will likely continue to play a role in clarifying what fuzzballs and firewalls really mean, and will likely also have an impact on how gauge/gravity correspondences are understood.

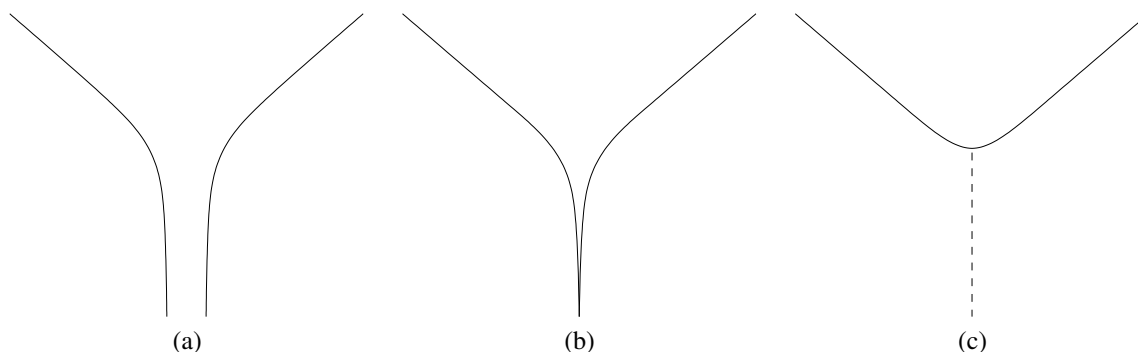


Figure 11. Schematic illustration of the class of geometries which consists of asymptotic region and (a) a macroscopic AdS_4 throat, (b) a small AdS_4 throat, and (c) semiclassically smooth capped of geometry, potentially accompanied by structures only visible when string corrections are included.

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